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PLASTIC CONCENTRATION FACTORS IN FLAT NOTCHED SPECIMENS OF AISI 4340 STEEL

RALPH PAPIRNO
THEORETICAL & APPLIED MECHANICS RESEARCH LABORATORY

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PLASTIC CONCENTRATION FACTORS IN FLAT NOTCHED SPECIMENS OF AISI 4340 STEEL

Technical Report by

RALPH PAPIRNO

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ABSTRACT

Various authors have proposed methods for predicting the plastic behavior at the root of a notch under monotonic loading. Among these is a method by Neuber, which was originally developed for shear but which has been empirically applied, at Neuber's suggestion, to tension and compression loading. There has been only a limited confirmation of Neuber's method in tests of notched specimens. Additional confirmation is given in this report for a range of notch geometry.

The basis of the Neuber approach is the suggested rule that the geometric mean of the stress and strain concentration factors, when the root of the notch is plastic, is given by the theoretical elastic concentration factor:

$$(K_\sigma K_\varepsilon)^{1/2} = K_t.$$

The Neuber rule is evaluated using an appropriate analytic representation of the stress-strain curve of AISI 4340 steel and predictions of maximum notch strain versus nominal net section stress are developed. The theoretical results, when compared with test data from flat notched specimens of the same material with a range of initial elastic concentration factors, show agreement within 5%. It is shown that the limitations of the strain gages in measuring the notch root strains can account for a major part of the discrepancy.

CONTENTS

LIST OF SYMBOLS

A_{net} = net section area, in.^{2}
a = exponent
b = notch specimen half gross-width; $b = w/2$, in.
C = general coefficient, strain-stress × strain power law
C_p = coefficient; $C = C_p$ in plastic region
C_t = coefficient; $C = C_t$ in transition region
D = coefficient, strain-stress power law
d = notch specimen net-section width, in.
E = modulus of elasticity, psi
K_t = theoretical elastic concentration factor
K_ϵ = strain concentration factor, elastic or plastic
K_σ = stress concentration factor, elastic or plastic
K_∞ = elastic concentration factor for a notched semi-infinite plate
ℓ = notch length, in.
m = exponent; $p = m$ in transition region
n = exponent; $p = n$ in plastic region
P = applied load, lb.
p = general exponent, strain-stress × strain power law
q = exponent, strain-stress power law
r = notch root radius, in.
t = specimen thickness, in.
w = notch specimen gross width, in.
α = exponent
β = exponent
ϵ = strain, in./in.
ϵ_e = elastic strain, in./in.
ϵ_n = maximum notch strain, in./in.
ϵ_0 = nominal net-section strain, in./in.
ϵ_p = plastic strain, in./in. (See Eq 8)
ϵ_{pl} = proportional limit strain, in./in.
ϵ_t = transitional strain, in./in. (See Eq 7)
ϵ_y = 0.1% offset yield strain, in./in.
σ = stress, psi
σ_e = elastic stress, psi
σ_n = maximum notch stress, psi
σ_0 = nominal net-section stress, psi; $\sigma_0 = P/A_{net}$
σ_p = plastic stress, psi (See Eq 8)
σ_t = transitional stress, psi (See Eq 7)

INTRODUCTION

Methods of predicting the plastic behavior in notches and other discontinuities under cyclic loading have been developed by adopting static prediction methods for low cycle fatigue. A study by Stowell,[1] of the plastic concentration factors around a hole in a plate under static loading, was generalized by Hardrath and Ohman[2] to include various other geometric discontinuities and then applied by Crews and Hardrath[3] to cyclic loading. Kuhn and Figge,[4] analogously building on earlier work of Neuber,[5] were also able to arrive at a scheme for predicting the strength of notched parts under cyclic loading. As another example of this process, Wetzel,[6] using a later formulation of Neuber[7] on plastic concentration factors under monotonic loading, was able to develop a method for relating the conditions in a smooth specimen to those in a notched specimen under cyclic loading. The aforementioned examples are not meant to be an exhaustive survey of the field but have been presented as illustrative examples of a particular approach to cyclic, plastic behavior of elements with stress concentrations. These approaches have in common a particular sequence of analysis:

a. Modification of the conventional, monotonic loading elastic concentration factor to take into account plastic behavior in the notch or other discontinuity.

b. Experimental verification of the derived plastic concentration factor under monotonic loading conditions.

c. Modification of the plastic concentration factor for cyclic loading to obtain fatigue concentration factors.

d. Experimental verification of the values of the fatigue reduction factors.

A crucial step in this described sequence is the development of a well-founded method of predicting plastic concentration factors under monotonic loading for later application to low cycle fatigue. In monotonic loading, the plastic concentration factor formulation is evaluated using the virgin stress-strain curve of the material. This formulation then becomes the basis for fatigue behavior predictions when the cyclic stress-strain curve is substituted for the virgin stress-strain curve to develop the analytic results.

The method developed by Neuber[7] for predicting plastic concentration factors is attractive since it can easily be adapted to machine computation. Because it has had only limited experimental confirmation, the study described here was undertaken to assess its predictive value for monotonic loading, prior to applying the theory in low cycle fatigue. This report describes a combined analytical and experimental investigation with the following major objectives:

a. To refine the procedure of plastic concentration factor prediction for monotonic loading using an appropriate analytic representation of the virgin stress-strain curve of AISI 4340 steel.

To perform experiments on notched tension specimens of AISI 4340 steel with a range of initial elastic concentration factors for comparison with theoretical prediction of the plastic concentration factors resulting from monotonic loading.

NEUBER FORMULATION

The basis of Neuber's approach is the suggested rule that the geometric mean of the stress and strain concentration factors is the theoretical elastic concentration factor:

$$(K_\sigma K_\varepsilon)^{1/2} = K_t \tag{1}$$

where $K_\sigma = \sigma_n/\sigma_0$ and $k_\varepsilon = \varepsilon_n/\varepsilon_0$ for plane stress.

Although the original formulation of Neuber's rule was developed for monotonic loading in shear, Neuber has suggested and there has been some experimental evidence to show that it may also apply to tension or compression loading. Krempl[8] presented plastic strain and stress concentration factors for notched specimens (nominal $K_t = 3$) of carbon steel, 2.5 Cr-1 Mo alloy steel, and type 304 stainless steel from which geometric means could be computed. The calculated discrepancy between the geometric mean of the plastic stress and strain concentration factors and the theoretical elastic concentration factor averaged less than approximately $\pm$ 5%. These results should not be considered conclusive since the precise values of K_t for each of the individual specimens was not reported and data points were taken from charts. Nevertheless, Krempl's results were sufficiently good to warrant further pursuit of the Neuber approach.

It is possible to rewrite Equation 1 by applying the definition of strain and stress concentration factors referred to the nominal net section stress and strain as

$$(\sigma_n \varepsilon_n / \sigma_0 \varepsilon_0)^{1/2} = K_t \tag{2}$$

or

$$(\sigma_0 \varepsilon_0) K_t^2 = (\sigma_n \varepsilon_n). \tag{3}$$

The left-hand side of Equation 3 refers to the nominal net section stress and strain (subscript 0) while the right-hand side refers to conditions at the root of the notch (subscript n). In order to determine K_σ or K_ε from Equation 3, it is necessary that the equation be expressed in terms of stress for the former or in terms of strain for the latter. This is most easily done by the use of a power law of stress-strain behavior such as

$$\varepsilon = D\sigma^q. \tag{4}$$

It is shown later in this report that calculations are facilitated if Equation 4 is transformed into

$$\varepsilon = C(\sigma\varepsilon)^p \tag{5}$$

where $C = D^{[1(q + 1)]}$ and $p = q/(q + 1)$.

In the next section of this report, the stress-strain properties of heat-treated AISI 4340 steel are given in the form of Equation 5 by using a curve-fitting procedure on test data.

ANALYTIC STRESS-STRAIN RELATIONS FOR AISI 4340 STEEL

A graph of ε versus $(\sigma\varepsilon)$ for heat treated AISI 4340 steel plotted on logarithmic coordinates reveals two linear regions in addition to the elastic region as shown schematically in the upper graph of Figure 1. The linear region between the proportional limit strain ε_{pl} and the 0.1% offset yield strain ε_y has been designated here as the *transitional* region and corresponds to the knee of the conventional stress-strain curve shown in the lower graph of Figure 1. The region where strains are in excess of this yield strain has been designated here as the *plastic* region.

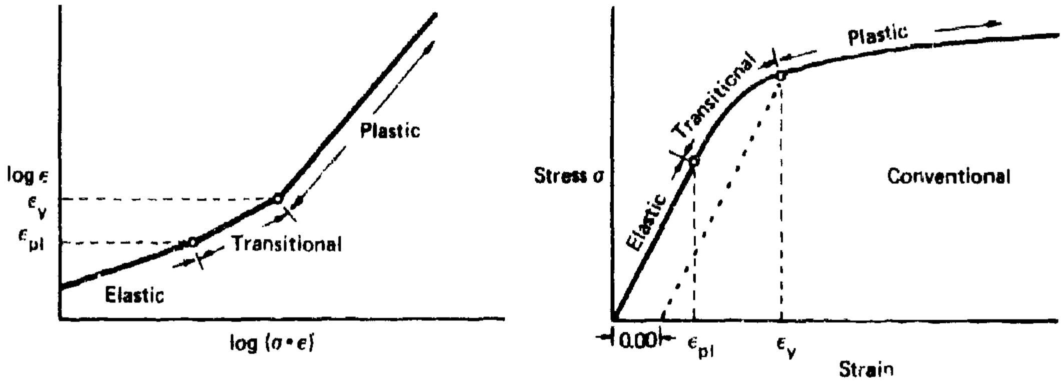

Figure 1. Log stress-log stress-strain and stress-strain curves for AISI 4340 steel (schematic)

All three regions can be represented by equations of the form given in (5):

Elastic $\qquad \varepsilon_e = (1/E)^{1/2}\ (\sigma_e\varepsilon_e)^{1/2}$ (6)

Transitional $\qquad \varepsilon_t = C_t\ (\sigma_t\varepsilon_t)^m$ (7)

Plastic $\qquad \varepsilon_p = C_p(\sigma_p\varepsilon_p)^n.$ (8)

A typical log ε versus log $(\sigma\varepsilon)$ plot for one heat of heat-treated AISI steel is shown in Figure 2. An automated data reduction procedure using a least-squares analysis was applied to obtain the coefficients and exponents shown in the figure. This procedure involved a number of steps: (1) autographic recording of the engineering stress-strain curve; (2) automatic analog to digital conversion of the data on punched tape; (3) tape to card conversion; (4) computer data reduction using a specially written program which included linear and logarithmic least-squares analyses.

NOTCH STRESS AND STRAIN ANALYSIS

Referring to Equation 3, it should first be noted that the σ_0 and ε_0 are nominal values referring to the net section. The net section conditions are defined as follows: $\sigma_0 = P/A_{net}$ and ε_0 is given by an equation of the form of Equation 5.

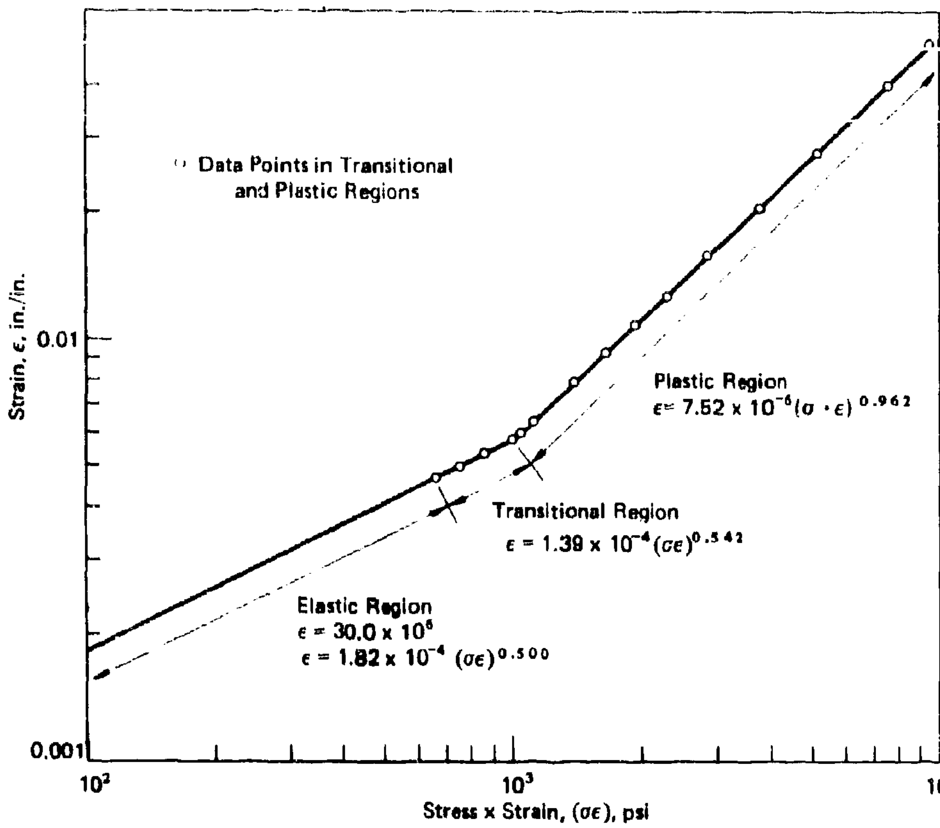

Figure 2. Log stress-long stress-strain data for one heat of heat treated AISI 4340 steel showing material constants for three regions

The net section strain conditions will depend upon whether the net section stress is elastic, transitional, or plastic. Referring to (3), it is possible to recognize six cases of net section (right-hand side) and notch (left-hand side) conditions for any given notch geometry as determined by the value of the elastic concentration factor K_t. These are enumerated below:

Case No.	Net Section Condition	Notch Condition
1	Elastic	Elastic
2	Elastic	Transitional
3	Elastic	Plastic
4	Transitional	Transitional
5	Transitional	Plastic
6	Plastic	Plastic

A. Analytic Formulations

A given specimen with a predetermined value of the elastic concentration factor will progress through a number of cases as it is loaded monotonically to failure. The actual progression will depend on the magnitude of K_t and on the toughness of the test material. A particular specimen need not progress through all the cases as it is loaded to fracture. For example, a specimen with high K_t manufactured of a material with only moderate toughness may progress through cases 1, 2, and 3 only. With higher toughness the progress may be through 1, 2, 3, 5, and 6. Other combinations of toughness and K_t would lead to different progressions.

It is assumed in the following development that plane stress conditions in a flat notch specimen prevail and that no notch strengthening occurs:

Case 1: The elastic conditions of case 1 follow the familiar stress concentration factor relations where:

a) $K_\sigma = K_\epsilon = K_t$;

b) $\sigma_n = K_t \sigma_0$ and $\epsilon_n = K_t \epsilon_0$.

In the subsequent development of cases 2 through 6, the notch strain and stress values will be obtained by suitable substitutions of relations (6), (7), or (8), into (3). Then the appropriate plastic strain and stress concentration factors will be given. Each of the cases is considered separately below.

Case 2: Elastic Net Section - Transitional Notch ($\epsilon_0 < \epsilon_{pl}$; $\epsilon_n < \epsilon_y$)

a) Notch strain: Since the nominal net section is elastic the right-hand side of (3) may be expressed in terms of strain and the elastic modulus:

$$\sigma_n \epsilon_n = (\epsilon_0^2 E) K_t^2. \tag{9}$$

Now (9) can be combined with (7) and solved for ϵ_n

$$\epsilon_n = C_t (\epsilon_0^2 K_t^2 E)^m. \tag{10}$$

b) Notch Stress: The right-hand side of (3) may be expressed in terms of stress and the elastic modulus since:

$$\epsilon_0 = \sigma_0 / E \tag{11}$$

then

$$\sigma_n \epsilon_n = (\sigma_0^2 / E) K_t^2. \tag{12}$$

The right-hand side of (10) may also be expressed in terms of stress

$$\epsilon_n = C_t (\sigma_0^2 / E)^m K_t^{2m}. \tag{13}$$

Now (13) is substituted into (12) and solved for σ_n:

$$\sigma_n = (1/C_t)(\sigma_0^2 / E)^{(1-m)} K_t^{(2-2m)}. \tag{14}$$

c) Strain Concentration Factor:

By definition the strain concentration factor is given by

$$K_\varepsilon = \varepsilon_n/\varepsilon_0. \tag{15}$$

Substitution of (10) into (15) results in

$$K_\varepsilon = C_t(K_t^2E)^m\varepsilon_0^{(2m-1)}. \tag{16}$$

d) Stress Concentration Factor:

Analogously to (15) the stress concentration form is

$$K_\sigma = \sigma_n/\sigma_0. \tag{17}$$

Substitution of (14) into (17) results in

$$K_\sigma = (1/C_t)(K_t^2/E)^{(1-m)}(\sigma_0)^{(1-2m)} \tag{18}$$

Case 3: Elastic Net Section - Plastic Notch ($\varepsilon_0<\varepsilon_{pl}$; $\varepsilon_n>\varepsilon_y$)

This case is directly analagous to case 2; however, the material properties for the notch are described by (8). The equations can be written by inspection using (10), (14), (16), and (18) as guides.

a) Notch strain (analogous to (10)):

$$\varepsilon_n = C_p(\varepsilon_0^2K_tE)^n. \tag{19}$$

b) Notch Stress (analogous to (14)):

$$\sigma_n = (1/C_p)(\sigma_0^2/E)^{(1-n)}K_t^{(2-2n)}. \tag{20}$$

c) Strain Concentration Factor (analogous to (16)):

$$K_\varepsilon = C_p(K_t^2E)^{(n)}\varepsilon_0^{(2n-1)} \tag{21}$$

d) Stress Concentration Factor (analogous to (18)):

$$K_\varepsilon = (1/C_p)(K_t^2/E)^{(1-n)}(\sigma_0)^{(1-2n)}. \tag{22}$$

Case 4: Transitional Net Section - Transitional Notch ($\varepsilon_{pl}<\varepsilon_0<\varepsilon_n\leq\varepsilon_y$)

a) Notch Strain: This is quite simply obtained by substitution of (7) into both sides of (3), resulting in

$$(\varepsilon_n/C_t)^{1/m} = K_t^2(\varepsilon_0/C_t)^{1/m} \tag{23}$$

which can be reduced to

$$\varepsilon_n = K_t^{2m}\,\varepsilon_0. \tag{24}$$

b) Notch Stress: Substitutions of (24) into (3) results in an expression for notch stress:

$$\sigma_n = K_t^{2-2m}\sigma_0. \tag{25}$$

c) Strain Concentration Factor: This is obtained directly from (24):

$$K_\varepsilon = K_t^{2m}. \tag{26}$$

d) Stress Concentration Factor: This is obtained directly from (25):

$$K_\sigma = K_t^{2-2m}. \tag{27}$$

Case 5: Transitional Net Section - Plastic Notch ($\varepsilon_{pl}<\varepsilon_0\leq\varepsilon_y$; $\varepsilon_y<\varepsilon_n$)

a) Notch Strain: Determination of notch strain in this case is made by substitution of (7) into the right-hand side of (3) and (8) into the left-hand side. The result, after simplification is:

$$\varepsilon_n = K_t^{2n}C_p(\varepsilon_0/C_t)^{n/m} \tag{28}$$

b) Notch Stress:

By suitable algebraic manipulation of (7) and (8) and subsequent substitution into (28), the strain factors can be transformed to stress with the following result:

$$\sigma_n = K_t^{2-n}(1/C_p)C_t^{\alpha}\sigma_0^{\alpha} \tag{29}$$

where $\alpha = (1-n)/(1-m)$.

c) Strain Concentration Factor: Substitution of (28) into (15) results in

$$K_\varepsilon = K_t^{2n}C_p(1/C_t)^{n/m}\varepsilon_0^{(n-m)/m}. \tag{30}$$

d) Stress Concentration Factor: Substitution of (29) into (17) results in

$$K_\sigma = K_t^{(2-2n)}(1/C_p)(C_t^{\alpha})\sigma_0^{(\alpha-1)}. \tag{31}$$

Case 6: Plastic Net Section - Plastic Notch ($\varepsilon_y<\varepsilon_0<\varepsilon_n$)

This case is directly analogous to case 4 and the various relations can be written by inspection using the appropriate material constants for the plastic range.

a) Notch Strain:

$$\epsilon_n = K_t^{2n} \epsilon_0. \tag{32}$$

b) Notch Stress:

$$\sigma_n = K_t^{2-2n}\sigma_0. \tag{33}$$

c) Strain Concentration Factor:

$$K = K_t^{2n}. \tag{34}$$

d) Stress Concentration Factor:

$$K_\sigma = K_t^{2-2n}. \tag{35}$$

B. Calculation of Theoretical Results

Laborious computations are required to evaluate the stress and strain history of a particular specimen as it is monotonically loaded and progresses through the various cases. A computer program was developed to perform the calculations for any specimens of a given K_t value, which properly discriminates the correct progression through the various cases and eliminates those which are unnecessary. The program yields the notch stress and strain values as a function of net section stress and strain, the plastic concentration factors, and the required loads for a given net section area. A listing of the program and a typical printout are given in Appendix A.

As an added convenience, a program for computing the elastic concentration factor K_t can also be developed and combined with the plastic program. Such a program for a flat tension specimen with semicircular notch ends was developed using separate formulations for deep and shallow notches given subsequently in this paper in (43-46). The combined program is also given in Appendix A.

In the experiments, described in the next section of this report, notched tension specimens were loaded and an autographic record of net section stress versus maximum notch strain was obtained. The theoretical values of the two parameters were obtained from the computer program directly without the necessity of using an explicit relation between notch strain and net section stress of the form:

$$\sigma_0 = f(\epsilon_n). \tag{36}$$

For completeness, however, the explicit relations between net section stress and notch strain have been developed for each of the six cases and they are listed below in (37-42).

Case 1: $\sigma_0 = (E/K_t)\epsilon_n$. (37)

Case 2: $\sigma_0 = E^{1/2}(\epsilon_n/C_t K_t)^{1/2m}$ for $\epsilon_0 \le \epsilon_{pl} < \epsilon_n < \epsilon_y$. (38)

Case 3: $\sigma_0 = E^{1/2}(\epsilon_n/C_p K_t)^{1/2n}$ for $\epsilon_0 \le \epsilon_{pl} \epsilon_n > \epsilon_y$ (39)

Case 4: $\sigma_0 = (1/K_t^{(2-2m)} C_t^{1/m})\epsilon_n^{(1-m)/m}$ for $\epsilon_{pl} < \epsilon_0 < \epsilon_n < \epsilon_y$. (40)

Case 5: $\sigma_0 = (\epsilon_n/K_t^{2n} C_p C_t^{\beta})^{(1-m)/m}$ for $\epsilon_{pl} < \epsilon_0 \le \epsilon_y < \epsilon_n$ (41)

where $\beta = [1-n(1-m)]/m(1-m)$

Case 6: $\sigma_0 = (1/K_t^{(2-2n)})(\epsilon^{(1-n)}/C_p)^{1/n}$ for $\epsilon_y < \epsilon_0 < \epsilon_n$ (42)

EXPERIMENTAL PROCEDURE

Externally notched flat tension specimens with the notch configurations shown in Figure 3 were fabricated from two lots of AISI 4340 steel plate. These were given identical heat treatments before specimen fabrication. Electrical resistance strain gages were installed at the roots of the notches and the specimens were monotonically loaded to fracture. The nominal net section stress (converted from applied load) and the notch maximum strain were autographically recorded up to approximately 2% strain. Monotonic loading was continued until fracture and the fracture load was recorded. Stress-strain properties were obtained from standard flat tension specimens using strain gages and clip-on extensometers. The stress-strain data were recorded autographically and the autographic records were analyzed using an automated data reduction process. The details of the procedure are described in this section; comparison of the experimental results with the theoretical predictions is given in the next section.

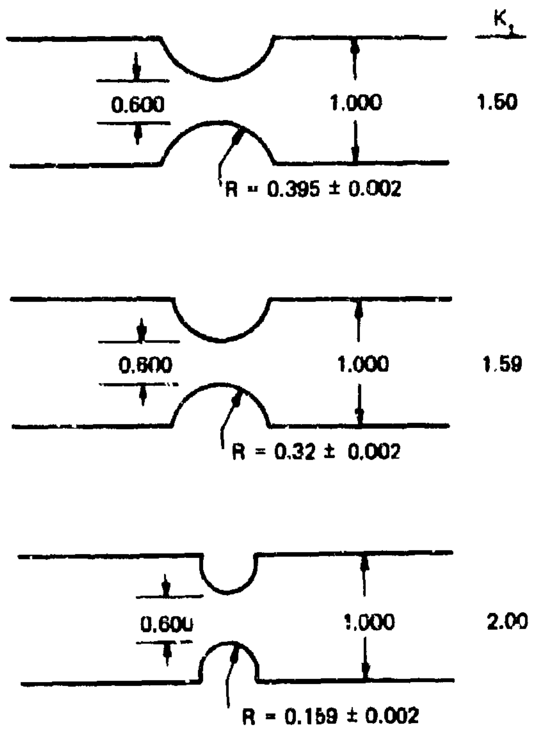

Figure 3. Notch configurations and notch dimensions of 0.1-inch thick flat notched specimens.

A. Material

Material properties specimens and notched specimens were fabricated from two separate heats of AISI 4340 steel plate, received in the annealed condition. Lot No. 1, used for one notched specimen and

one smooth tension specimen, was received as 0.5-inch-thick plate while Lot No. 2 was received as 0.75-inch-thick plate. The chemical analyses of both lots are given below together with the heat treatment details:

Chemical Analyses (wt %)

	C	Mn	P	S	Si	Ni	Cr	Mo	Fe
Lot No. 1, Heat 3931362	0.40	0.74	0.003	0.004	0.22	1.88	0.87	0.25	Remainder
Lot No. 2, Heat 3830298	0.39	0.80	0.005	0.006	0.23	1.77	0.78	0.26	Remainder

Heat Treatment (Applied Mechanics Research Laboratory Designation: A-16)

Austenitize at 2300 F, 1 Hr; Furnace Cool to 1550 F
Oil Quench to R. T.; Hold 15 min.
Double Normalize in Salt at 1650 F, 1 hr; Air Cool
Reaustenitize in Salt at 1550 F, 1 hr
Oil Quench to R.T.; Hold 15 min.
Quench to Liquid Nitrogen Temperature
Temper in Salt at 920 F, 1 hr
Water Quench to R.T.

The as-received material was cut into blanks from which one or more specimens could later be prepared and heat treated in its full thickness. The thickness was then reduced to 0.10 inch for specimen preparation.

B. Stress-Strain Tests

Standard flat + tension specimens (2-inch gage length, 0.50-inch wide, and 0.10-inch thick) were tested to obtain stress-strain properties of the heat treated material. These were loaded in a Tinius Olsen hydraulic testing machine and strains were measured either by electrical resistance strain gages or by a clip-on extensometer. The data were recorded autographically on a X-Y recorder whose axes were calibrated for each specimen to read strain and stress directly (rather than load and extension). The resulting stress-strain data were automatically converted to digital form and subsequently were analyzed by computer using least-squares analyses: Linear, in the elastic region to obtain E; and linear-logarithmic in the nonelastic regions to obtain the material constants and exponents required for the experimental approximation of the stress-strain properties.

C. Notch Specimen Preparation and Testing

For the design of the notched specimen shown in Figure 3 with K_t values of 1.5, 1.59, and 2.00, the following relations, empirically derived by Heywood,[9] were employed:

$$K_t = [(\ell/r)/(1.55[w/d]-1.3)]^a \quad (43)$$

where

$$a = [w/d - 1 + 0.5(\rho/r)^{1/2}]/[(w/d) - 1 + (\ell/r)^{1/2}] \qquad (44)$$

(See Figure 4 for identification of notch parameters.)

These relations were incorporated into a computer program. In the interest of completeness, a formulation for specimens where $K_t > 2$ was also included in the program. Baratta and Neal[10] using a prior formulation of Bowie[11] showed that the following relations are appropriate for deeper U notches:

$$K_t = [1 + 0.182(\ell/b) - 1.071(\ell/b)^2 + 1.727(\ell/b)^3][1 - (\ell/b)]K_\infty \qquad (45)$$

where

$$K_\infty = 0.775 + 2.243(\rho/r)^{1/2}. \qquad (46)$$

After manufacture, the specimens were carefully measured and the actual stress concentration factors for the notches were re-evaluated using the appropriate formulas. Because of manufacturing tolerances the values of K_t computed from actual dimensions could depart from the nominal values by several percent.

The basic notch specimen design is shown in Figure 5 for one notch configuration. This basic design was used for all the notches shown in Figure 3. Specimen blanks were cut from the as-received material, heat treated, and then reduced in thickness to 0.10-inch by machining equal amounts of material from each surface. The various holes and contours were machined into the final thickness blank.

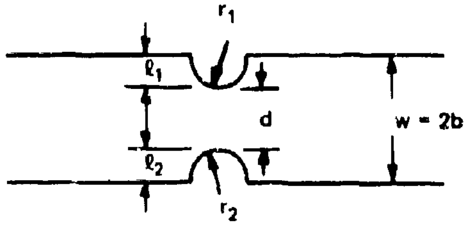

Figure 4. Identification of notch parameters

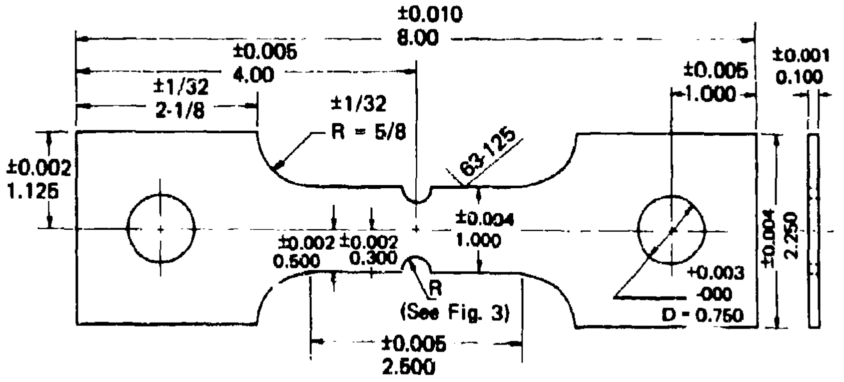

Figure 5. Notch specimen design and dimensions. Notch configuration dimension given in Figure 3.

Strain gages, BLH Type FAP-03-12, 0.040-inch long and 0.05-inch wide, were cemented at the notch roots, one in each notch in individual specimens. These were electrically connected in series so that bending components of strain were eliminated and so that the longitudinal strain reading obtained was the average for the two notches in each specimen. The specimen gages formed one arm of a Wheatstone bridge with compensating gages on a dummy specimen forming an opposite arm. The remainder of the bridge consisted of precision resistors. The unbalance bridge voltage was recorded on a Hewlett-Packard X-Y recorder calibrated to read 0.001-in./in. strain per one-half inch of pen displacement along the recorder X-axis.

The bridge energizing voltage was held to approximately two volts. This limited the power dissipation of the gages to less than 5 watts/sq in., a sufficiently small value so that excessive heating of the specimen in the notch root was avoided.

Loads were recorded on the Y-axis of the recorder. By taking the specimen area into account it was possible to calibrate the recorder to read net section stress directly on a scale where 10 ksi was the equivalent of one-half inch of pen displacement. The resulting autographic recording showed net section stress as a function of notch strain.

EXPERIMENTAL AND THEORETICAL RESULTS

In a test of a notched specimen it is possible to measure the maximum strain at the root of the notch and to determine the net section stress from the net section area and the applied load. Theoretical values of these two parameters can also be developed. It is assumed that a comparison of the theoretical and experimental values of the two parameters will constitute a valid test of Neuber's hypothesis although the hypothesis itself is stated in slightly different terms.

A. Mechanical Property Data

The reduced material property data for the two separate heats of material used for specimen manufacture are given in Table I. The constants given for transition and plastic regions of the stress-strain curve are those applicable to Equations 7 and 8.

Table I. STRESS-STRAIN DATA FOR TWO HEATS OF HEAT-TREATED AISI 4340 STEEL OBTAINED FROM A LEAST-SQUARES, CURVE-FITTING ANALYSIS

Heat No.	Elastic Modulus, 10^6 psi	ε_{pl} %	ε_y %	Transition-Region C_t	m	Plastic-Region C_p	n
1	30.0	0.47	0.60	1.39×10^{-4}	0.542	7.52×10^{-6}	0.962
2	29.6	0.53	0.65	0.65×10^{-4}	0.655	7.50×10^{-6}	0.962

The values of ε_{pl} and ε_y were calculated from the fitted curves. The proportional limit strain ε_{pl} has been taken at the intersection of the elastic line with the curve representing the transition region. The yield strain value, ε_y, represents the intersection of the transition region curve with the plastic region curve.

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Key Words

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Stress concentration
Notch strength

The basis of a method by Neuber for predicting plastic concentration factors is the suggested rule that the geometric mean of the stress and strain concentration factors is given by the theoretical elastic concentration factor:

$$(K_\sigma K_\epsilon)^{1/2} = K_t.$$

The Neuber rule is evaluated using an appropriate analytic representation of the stress-strain curve of AISI 4340 steel and predictions of maximum notch strain versus nominal net section stress are developed. The theoretical results, when compared with test data from flat notched specimens of the same material with a range of initial elastic concentration factors, show agreement within 5%. It is shown that the limitations of the strain gages in measuring the notch root strains can account for a major part of the discrepancy.

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Army Materials and Mechanics Research Center,
Watertown, Massachusetts 02172
PLASTIC CONCENTRATION FACTORS IN
FLAT NOTCHED SPECIMENS OF AISI
4340 STEEL - Ralph Papirno

Technical Report AMMRC TR 70-2, February 1970, 24 pp - illus,
tables, appendix, D/A Project 1T061102B32A,
AMCMS CODE 501B.11.855

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B. Notch Specimen Data

Experimental and theoretical net section stress and notch strain data for a typical specimen are given in Figure 6. The figure shows fairly good agreement between experiment and theory up to approximately 1.4% strain which is typical for all specimens tested. A summary of the comparison between experiment and theory for all the tests is given in Table II below. The agreement in the elastic region was within two percent.

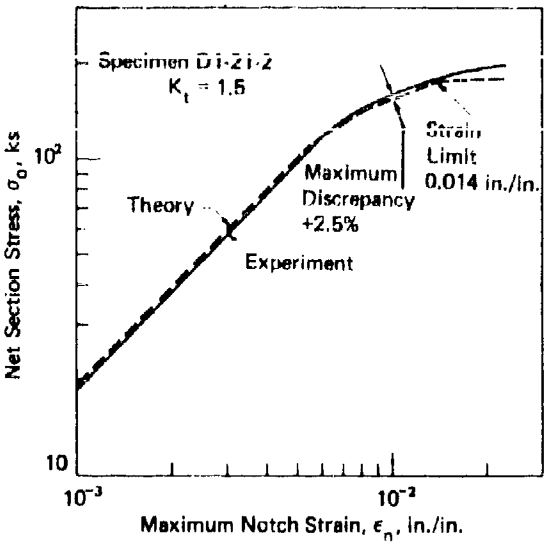

Figure 6. Typical experimental and theoretical data for K_t = 1.5

In each case the experimental values of notch stress for a given notch strain were greater than the theoretically predicted values. Beyond the indicated strain limit values shown in the table there was a much larger discrepancy which is interpreted as an effect of multiaxial stress and resulting notch strengthening. It is not altogether clear whether the discrepancies shown in Table II are also a result of multiaxial stress effects or result from inevitable variations in heat treating in the separate batches which were used in the program. With the exception of one specimen, the theory appears to be conservative by approximately 5% up to about 1.5% notch strain. However, certain possible errors, discussed below, could account for the discrepancy.

C. Experimental Accuracy

Load errors (and hence net section stress errors) were neglible since the testing machine was calibrated just prior to the testing program using proving rings whose own calibration was traceable to the Bureau of Standards.

Table II. COMPARISON BETWEEN EXPERIMENTAL AND PREDICTED NET SECTION STRESS-NOTCH STRAIN DATA

Specimen No.	Heat No.	K_t	Discrepancy Exp. vs Theory	Strain Limit in./in.
A-1-1	1	1.59	<1%	0.0200
D1-21-4	2	1.50	+4.0%	0.0140
D1-21-2	2	1.50	+2.5%	0.0140
D13-12-9	2	1.50	+4.8%	0.0140
D1-31-1	2	2.00	+6.5%	0.0140
D13-13-5	2	2.0	+5.5%	0.0160
D13-13-9	2	2.0	+4.5%	0.015

Major sources of error in the strain measurements resulted from the following:

1. The manufacturer's stated ±3% uncertainty in the value of the gage factor.

2. The presence of a strain gradient at the notch root with the maximum strain value confined in an area smaller than that of the strain gage.

3. An estimated possible ±0.01 inch deviation of the position of the center line of the gage from the center of the notch root.

The magnitude of the latter two sources of error is not known, however, both would tend to produce strain readings which were less than the actual maximum strain in the notch.

Some uncertainty in the results arises from the fact that it was not possible to heat treat the stress-strain specimens and the notch specimens all in the same batch because of the limited capacity of the heat-treating facilities. Tests indicated that there could be a variation in the computed notch strains of ±1% based upon scatter of the material properties.

CONCLUSIONS

The major conclusions of this study are as follows:

1. Predictions of plastic notch strain and notch stress can be developed using Neuber's rule and an analytic representation of the stress-strain curve in three regions: elastic, transitional, and plastic.

2. Theoretical predictions of net section stress versus plastic notch maximum strain are within 5%, on the average, of experimentally observed values for notched specimens of heat-treated AISI 4340 steel up to a maximum notch strain value of approximately 0.015 in./in.

ACKNOWLEDGMENTS

A number of staff members of the Theoretical and Applied Mechanics Research Laboratory contributed to the investigation described herein. Mr. John Campo performed the majority of the laboratory tests; Mr. John Hannon delicately applied the strain gages in the notch roots and contributed to the development of the automated data analysis program; Mr. Joseph Wong, a summer student aide, assisted with the computer programming and with the calculations. I am grateful to all who contributed and am happy to acknowledge their contributions.

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APPENDIX A - COMPUTER PROGRAMS

The programs are written as Fortran I

1. Program No. 1. Plastic stress and strain in notched tension specimens with a given value of the elastic concentration factor and given material properties. The input data required are given below:

Parameter	Fortran Designation
Elastic Concentration Factor, K_t	X
Modulus of Elasticity, E, psi	E
Transitional Exponent, m	Q1
Plastic Exponent, n	Q2
Transitional Coefficient, C_t	C1
Plastic Coefficient, C_p	C2
Proportional Limit Strain, ϵ_{pl}, percent	EPL
0.1% Offset Yield Strain, ϵ_y, percent	EY
Program Cut-Off Notch Strain, percent	EMAX
Specimen Net-Section Area, sq in.	AREA

Specimen number is entered using up to 11 alphanumeric characters.

A listing of the program is given on page 17.

2. Typical Output of Program No. 1. A typical output run of Program No. 1 for a specimen of AISI 4340 steel with and elastic concentration factor, K_t=1.5, is given on page 19.

3. Program No. 2. Elastic concentration factor and plastic stress and strain in notched tension specimens with given notch dimensions and given material properties.

This program combines a program for computing the elastic concentration factor using either of two methods (Heywood method for shallow semicircular notches or Bowie method for deep semicircular notches) with Program No. 1.

In addition to the material property parameters listed as input for Program No. 1, the following input data are required for the combined program:

Parameter	Fortran Designation
Specimen Width, w, in.	W
Length of Notch No. 1, ℓ_1, in.	A1
Length of Notch No. 2, ℓ_2, in.	A2
Radius of Notch No. 1, r_1, in.	R1
Radius of Notch No. 2, r_2, in.	R2
Specimen Thickness, t, in.	T

The method of computation is designated by the entry of NN in the 4th statement of the program lists given on page 20. For NN = 00, the Heywood method is used; for NN = 01, the Bowie method is used. The words "Heywood" or "Bowie" are entered in the same statement. The specimen number is entered as described previously for Program No. 1.

80 COLUMN PRINTOUT OF PROGRAM NO. 1

```
  1     READ 777,ITEST,P1,P2,P3,P4,P5,P6,P7,P8,P9,P10,P11
        IF(ITEST) 999,999,2
  2     PRINT 778, P1,P2,P3,P4,P5,P6,P7,P8,P9,P10,P11
        READ 889,Q1,Q2,C1,C2,EPL,EY,EMAX,F
        READ 101, X, AREA
        PRINT 102,E,EPL,EY,EMAX
        PRINT 103,C1,Q1,C2,Q2
        PRINT 112,X,AREA
        PRINT 13
        PRINT 14
        PRINT 15
        PRINT 16
        PRINT 202
        SO=20000.
        E=E*(10.**6)
 17     EO=SO/F
        EN=X*EO
        IF(EN-EPL)18,18,300
 18     SN=X*SO
        P=SO*AREA
        SOP=SO/1000.
        EOP=EO*100.
        SNP=SN/1000.
        ENP=EN*100.
        PRINT 222,P,EOP,SOP,ENP,SNP
        SO=SO+ 10000.
        GO TO 17
300     PRINT 302
301     SO=EO*E
        EN=C1*((X**2)*(EO**2)*E)**Q1
        IF(EN-EY) 19,19,400
 19     SN=(X**2)*(EO**2)*E/EN
        XS=SN/SO
        XE=EN/EO
        P=SO*AREA
        SOP=SO/1000.
        EOP=EO*100.
        SNP=SN/1000.
        ENP=EN*100.
        PRINT 222,P,EOP,SOP,ENP,SNP,XS,XE
        EO=EO+ 0.0001
        IF(EO-EPL) 301,301,500
400     PRINT 402
401     SO=EO*E
        EN=C2*(((X*EO)**2)*E)**Q2
        IF(EN-EMAX) 20,20,1
 20     SN=((X*EO)**2)*E/EN
        XS=SN/SO
        XE=EN/EO
        P=SO*AREA
        SOP=SO/1000.
        EOP=EO*100.
        SNP=SN/1000.
        ENP=EN*100.
        PRINT 222,P,EOP,SOP,ENP,SNP,XS,XE
        EO=EO+0.0001
        IF(EO-EPL) 401,401,600
500     PRINT 502
501     SO=((EO/C1)**(1./Q1))/EO
        EN=EO*X**(2.*Q1)
        IF(EN-EY) 21,21,600
 21     SN=((EO*SO)*X**2)/EN
        XS=SN/SO
        XE=EN/EO
        P=SO*AREA
        SOP=SO/1000.
        EOP=EO*100.
        SNP=SN/1000.
        ENP=EN*100.
        PRINT 222,P,EOP,SOP,ENP,SNP,XS,XE
        EO=EO+0.0005
        IF(EO-EY) 501,501,700
```

```
600     PRINT 602
 601    EN=C2*(X**(2.*Q2))*(EO/C1)**(Q2/Q1)
        IF (EN-EMAX) 22,22,1
 22     SO=((EO/C1)**(1./Q1))/EO
        SN=((EO*SO)*X**2)/EN
        XS=SN/SO
        XE=EN/EO
        P=SO*AREA
        SOP=SO/1000.
        ENP=EN*100.
        EOP=EO*100.
        SNP=SN/1000.
        PRINT 222,P,EOP,SOP,ENP,SNP,XS,XE
        EO=EO+0.0002
        IF(EO-EY)601,601,700
700     PRINT 702
 701    EN=EO*X**(2.*Q2)
        IF(EN-EMAX) 23,23,1
 23     SO=((EO/C2)**(1./Q2))/EO
        SN=((EO*SO)*X**2)/EN
        XS=SN/SO
        XE=EN/EO
        P=SO*AREA
        SOP=SO/1000.
        EOP=EO*100.
        SNP=SN/1000.
        ENP=EN*100.
        PRINT 222,P,EOP,SOP,ENP,SNP,XS,XE
        EO=EO+0.0002
        GO TO 701
 13     FORMAT(24X26HPREDICTED NOTCH PROPERTIES,//)
 14     FORMAT(5X HAPPL,ED,3X3HNOM,7X3HNOM,7X3HMAX,7X3HMAX,8X1HK,
      1 7X1HK)
 15     FORMAT(6X4HLOAD,5X6HSTRAIN,4X6HSTRESS,4X6HSTRAIN,4X6HSTRESS,
      1 4X3HSUB,5X3HSUB)
 16     FORMAT(5X6HPOUNDS,4X7HPERCENT,3X3HKSI,7X7HPERCENT,3X3HKSI,8X1HS,
      1 7X1HE,//)
 101    FORMAT(2F9.5)
 102    FORMAT(//,5X,2HE=,F6.2,6H*10**6,3X,4HEPL=,F8.6,3X,3HEY=,F8.6,3X,
      1 5HEMAX=,F8.6,//)
 103    FORMAT(5X,3HC1=,E11.5,5X,3HM1=,F7.5,5X,3HC2=,E11.5,5X,3HM2=,F7.5,
      1 //)

 112    FORMAT(10X6HK-AVE=,F9.5,10X9HNET AREA=,F9.5,1X5HSQ IN,//)
 202    FORMAT(/,20X15HELASTIC-ELASTIC,/)
 222    FORMAT(F12.4,F10.5,F9.1,F10.5,F7.1,3X,F7.4,2X,F7.4)
 302    FORMAT(/,20X20HELASTIC-TRANSITIONAL,/)
 402    FORMAT(/,20X15HELASTIC-PLASTIC,/)
 502    FORMAT(/,20X25HTRANSITIONAL-TRANSITIONAL,/)
 602    FORMAT(/,20X20HTRANSITIONAL-PLASTIC,/)
 702    FORMAT(/,20X15HPLASTIC-PLASTIC,/)
 777    FORMAT(I1,11A1)
 778    FORMAT(//,5X45HELASTIC, TRANSITION, AND PLASTIC ANALYSIS OF
      1 11HSPECIMEN NO,11A1,//)
 889    FORMAT (2F7.5,2E11.5,3F7.4,F6.2)
        GO TO 1
 999    STOP
        END
```

TYPICAL OUTPUT OF PROGRAM NO. 1

ELASTIC, TRANSITION, AND PLASTIC ANALYSIS OF SPECIMEN NO 01-21-[illegible]

E: 29.60*10**5   EPL: .005100   EY: .005500   EMAX: .025000

C1: .65000-04   B1: .65500   C2: .75000-05   B2: .96200

K-AVE: 1.50000   NET AREA .06000 SQ IN

PREDICTED NOTCH PROPERTIES

APPLIED LOAD POUNDS	NOM STRAIN PERCENT	NOM STRESS KSI	MAX STRAIN PERCENT	MAX STRESS KSI	K SUB S	K SUB E
		ELASTIC-ELASTIC				
1200.0000	.06757	20.0	.10135	30.0		
1800.0000	.10135	30.0	.15203	45.0		
2399.9999	.13514	40.0	.20270	60.0		
3000.0000	.16892	50.0	.25338	75.0		
3600.0000	.20270	60.0	.30405	90.0		
4199.9999	.23649	70.0	.35473	105.0		
4800.0000	.27027	80.0	.40541	120.0		
5399.9999	.30405	90.0	.45608	135.0		
5999.9999	.33784	100.0	.50676	150.0		
		ELASTIC-TRANSITIONAL				
6599.9999	.37162	110.0	.56770	162.0	1.4729	1.5276
6777.5998	.38162	113.0	.58780	165.0	1.4608	1.5403
6955.1997	.39162	115.9	.60806	168.0	1.4491	1.5527
7132.7996	.40162	118.9	.62848	170.9	1.4376	1.5648
7310.3995	.41162	121.8	.64906	173.8	1.4269	1.5768
		ELASTIC-PLASTIC				
7487.9993	.42162	124.8	.67957	174.5	1.3980	1.6094
7665.5991	.43162	127.8	.70998	174.8	1.3681	1.6447
7843.1992	.44162	130.7	.74196	175.1	1.3394	1.6798
8020.7991	.45162	133.7	.77452	175.4	1.3120	1.7150
8198.3989	.46162	136.6	.80785	175.7	1.2857	1.7500
8375.9987	.47162	139.6	.84156	176.0	1.2605	1.7850
8553.5987	.48162	142.6	.87654	176.2	1.2363	1.8200
8731.1987	.49162	145.5	.91189	176.5	1.2130	1.8549
8908.7985	.50162	148.5	.94791	176.8	1.1907	1.8897
9086.3984	.51162	151.4	.98460	177.1	1.1692	1.9245
9263.9984	.52162	154.4	1.02196	177.3	1.1484	1.9592
		TRANSITIONAL-PLASTIC				
9390.4171	.53162	156.5	1.05466	177.5	1.1344	1.9835
9574.0655	.55162	159.6	1.11324	177.9	1.1149	2.0181
9756.1757	.57162	162.6	1.17302	178.3	1.0964	2.0521
9934.5081	.59162	165.6	1.23379	178.6	1.0789	2.0854
10110.0086	.61162	168.5	1.29553	179.0	1.0622	2.1182
10282.8129	.63162	171.4	1.35822	179.3	1.0463	2.1504
		PLASTIC-PLASTIC				
10451.5553	.65162	174.2	1.42166	179.6	1.0313	2.1817
10464.0429	.67162	174.4	1.46529	179.9	1.0313	2.1817
10476.1796	.69162	174.6	1.50893	180.1	1.0313	2.1817
10487.9829	.71162	174.8	1.55256	180.3	1.0313	2.1817
10499.4727	.73162	175.0	1.59620	180.5	1.0313	2.1817
10510.6634	.75162	175.2	1.63983	180.7	1.0313	2.1817
10521.5724	.77162	175.4	1.68346	180.8	1.0313	2.1817
10532.2129	.79162	175.5	1.72710	181.0	1.0313	2.1817
10542.5985	.81162	175.7	1.77073	181.2	1.0313	2.1817
10552.7405	.83162	175.9	1.81437	181.4	1.0313	2.1817
10562.6522	.85162	176.0	1.85800	181.6	1.0313	2.1817
10572.3416	.87162	176.2	1.90164	181.7	1.0313	2.1817
10581.8200	.89162	176.4	1.94527	181.9	1.0313	2.1817
10591.0961	.91162	176.5	1.98891	182.0	1.0313	2.1817
10600.1793	.93162	176.7	2.03254	182.2	1.0313	2.1817
10609.0760	.95162	176.8	2.07617	182.4	1.0313	2.1817
10617.7971	.97162	177.0	2.11981	182.5	1.0313	2.1817
10626.3460	.99162	177.1	2.16344	182.6	1.0313	2.1817
10634.7309	1.01162	177.2	2.20708	182.8	1.0313	2.1817
10642.9583	1.03162	177.4	2.25071	182.9	1.0313	2.1817
10651.0337	1.05162	177.5	2.29435	183.1	1.0313	2.1817
10658.9632	1.07162	177.6	2.33798	183.2	1.0313	2.1817
10666.7513	1.09162	177.8	2.38162	183.3	1.0313	2.1817
10674.4041	1.11162	177.9	2.42525	183.5	1.0313	2.1817
10681.9254	1.13162	178.0	2.46888	183.6	1.0313	2.1817

80 COLUMN PRINTOUT OF PROGRAM NO. 2

```
1     READ 777,ITEST,P1,P2,P3,P4,P5,P6,P7,P8,P9,P10,P11
      IF(ITEST) 999,999,2
2     PRINT 778, P1,P2,P3,P4,P5,P6,P7,P8,P9,P10,P11
      READ 886,NN,Z2,Z3,Z4,Z5,Z6,Z7,Z8,Z9,Z10
      PRINT 900,Z2,Z3,Z4,Z5,Z6,Z7,Z8,Z9,Z10
      READ 888,W,  A1,A2,R1,R2,T
      READ 889,Q1,Q2,C1,C2,EPL,EY,EMAX,E
      B=W/2.
      AB1=A1/B
      AB2=A2/B
      AR1=A1/R1
      AR2=A2/R2
      D1=(B-A1)*2.
      D2=(B-A2)*2.
      DW1=D1/W
      DW2=D2/W
      D=W-A1-A2
5     IF(NN)6,6,7
6     U1=(1.-DW1+0.5*DW1*SQRT(AR1))/(1.-DW1+DW1*SQRT(AR1))
      X1=1.+(DW1*AR1/(1.55-1.3*DW1))**U1
      U2=(1.-DW2+0.5*DW2*SQRT(AR2))/(1.-DW2+DW2*SQRT(AR2))
      X2=1.+(DW2*AR2/(1.55-1.3*DW2))**U2
      GO TO 8
7     XINF= 0.78 + 2.243*SQRT(AR1)
      X1=(1.+0.182*AB1-1.07*AB1**2+1.727*AB1**3)*(1.-AB1)*XINF
      XINF2= 0.78 +2.243*SQRT(AR2)
      X2= (1.+0.182*AB2-1.07*AB2**2+1.727*AB2**3)*(1.-AB2)*XINF2
8     X=(X1+X2)/2.
      AREA=T*D
      PRINT 10, AB1,AR1,DW1,X1
      PRINT 11, AB2,AR2,DW2,X2
      PRINT 12,X,AREA
      PRINT 13
      PRINT 14
      PRINT 15
      PRINT 16
      PRINT 202
      S0=20000.
      F=E*(10.**6)
17    E0=S0/F
      EN=X*E0
      IF(EN-EPL)18,18,300
18    SN=X*S0
      P=S0*AREA
      SOP=S0/1000.
      EOP=E0*100.
      SNP=SN/1000.
      ENP=EN*100.
      PRINT 222,P,EOP,SOP,ENP,SNP
      S0=S0+ 10000.
      GO TO 17
300   PRINT 302
301   S0=E0*E
      EN=C1*((X**2)*(E0**2)*E)**Q1
      IF(EN-EY) 19,19,400
19    SN=(X**2)*(E0**2)*E/EN
      XS=SN/S0
      XE=EN/E0
      P=S0*AREA
      SOP=S0/1000.
      EOP=E0*100.
      SNP=SN/1000.
      ENP=EN*100.
      PRINT 222,P,EOP,SOP,ENP,SNP,XS,XE
      E0=E0+ 0.0001
      IF(E0-EPL) 301,301,500
400   PRINT 402
40    S0=E0*E
      EN=C2*(((X*E0)**2)*E)**Q2
      IF(EN-EMAX) 20,20,1
20    SN=((X*E0)**2)*E/EN
      XS=SN/S0
      XE=EN/E0
      P=S0*AREA
      SOP=S0/1000.
      EOP=E0*100.
      SNP=SN/1000.
      ENP=EN*100.
      PRINT 222,P,EOP,SOP,ENP,SNP,XS,XE
      E0=E0+0.0001
      IF(E0-EPL) 401,401,600
```

```
500     PRINT 502
 501    SO=((EO/C1)**(1./Q1))/EO
        EN=EO*K**(2.*Q1)
        IF(EN-EY) 21,21,600
  21    SN=((EO*SO)*K**2)/EN
        XS=SN/SO
        XE=EN/EO
        P=SO*AREA
        SOP=SO/1000.
        EOP=EO*100.
        SNP=SN/1000.
        ENP=EN*100.
        PRINT 222,P,EOP,SOP,ENP,SNP,XS,XE
        EO=EO+0.0005
        IF(EO-EY) 501,501,700
600     PRINT 602
 601    EN=C2*(K**(2.*Q2))*(EO/C1)**(Q2/Q1)
        IF (EN-EMAX) 22,22,1
 22     SO=((EO/C1)**(1./Q1))/EO
        SN=((EO*SO)*K**2)/EN
        XS=SN/SO
        XE=EN/EO
        P=SO*AREA
        SOP=SO/1000.
        ENP=EN*100.
        EOP=EO*100.
        SNP=SN/1000.
        PRINT 222,P,EOP,SOP,ENP,SNP,XS,XE
        EO=EO+0.0002
        IF(EO-EY)601,601,700
700     PRINT 702
 701    EN=EO*K**(2.*Q2)
        IF(EN-EMAX) 23,23,1
  23    SO=((EO/C2)**(1./Q2))/EO
        SN=((EO*SO)*K**2)/EN
        XS=SN/SO
        XE=EN/EO
        P=SO*AREA
        SOP=SO/1000.
        EOP=EO*100.
        SNP=SN/1000.
        ENP=EN*100.
        PRINT 222,P,EOP,SOP,ENP,SNP,XS,XE
        EO=EO+0.0002
        GO TO 701
   10   FORMAT(5X6HL1/R1=,F8.5,5X6HL1/R1=,F8.5,5X5HD1/D=,F8.5,
      1 5X,8HK-SUB-1=,F8.5,/)
   11   FORMAT(5X6HL2/D2=,F8.5,5X6HL2/R2=,F8.5,5X5HD2/D=,F8.5,
      1 5X8HK-SUB-2=,F8.5,/)
   12   FORMAT(10X6HK-AVE=,F9.5,10X9HNET AREA=,F9.5,1X5HSQ IN,//)
  13    FORMAT(24X26HPREDICTED NOTCH PROPERTIES,//)
   14   FORMAT(5X7HAPPLIED,3X5HNOM.,7X3HNOM,7X3HMAX,7X3HMAX,8X1HK,
      1 7X1HK)
   15   FORMAT(6X4HLOAD,5X6HSTRAIN,4X6HSTRESS,4X6HSTRAIN,4X6HSTRESS,
      1 4X3HSUB,5X3HSUB)
   16   FORMAT(5X6HPOUNDS,4X7HPERCENT,3X3HKSI,7X7HPERCENT,3X3HKSI,8X1HS,
      1 7X1HE,//)
202    FORMAT(/,20X15HELASTIC-ELASTIC,/)
  222   FORMAT(F12.4,F10.5,F9.1,F10.5,F7.1,3X,F7.4,2X,F7.4)
302    FORMAT(/,20X20HELASTIC-TRANSITIONAL,/)
402     FORMAT(/,20X15HELASTIC-PLASTIC,/)
502     FORMAT(/,20X25HTRANSITIONAL-TRANSITIONAL,/)
602     FORMAT(/,20X20HTRANSITIONAL-PLASTIC,/)
702     FORMAT(/,20X15HPLASTIC-PLASTIC,/)
 775    FORMAT(/A,5X45HELASTIC, TRANSITION, AND PLASTIC ANALYSIS OF     ,
      1 11HSPECIMEN NO,11A1,//)
 555    FORMAT(I1,9A1)
 888    FORMAT (6F8.4)
 889    FORMAT (2F7.5,2E11.5,3F7.4,F6.2)
 900    FORMAT(10X46HELASTIC CONCENTRATION FACTOR CALCULATED BY THE,
      1 9A1,6HMETHOD,/)
 777    FORMAT(I1,11A1)
 999    STOP
        END
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13. ABSTRACT Various authors have proposed methods for predicting the plastic behavior at the root of a notch under monotonic loading. Among these is a method by Neuber, which was originally developed for shear but which has been empirically applied, at Neuber's suggestion, to tension and compression loading. There has been only a limited confirmation of Neuber's method in tests of notched specimens. Additional confirmation is given in this report for a range of notch geometry.

The basis of the Neuber approach is the suggested rule that the geometric mean of the stress and strain concentration factors, when the root of the notch is plastic, is given by the theoretical elastic concentration factor: $(K_\sigma K_\epsilon)^{1/2} = K_t$.

The Neuber rule is evaluated using an appropriate analytic representation of the stress-strain curve of AISI 4340 steel and predictions of maximum notch strain versus nominal net section stress are developed. The theoretical results, when compared with test data from flat notched specimens of the same material with a range of initial elastic concentration factors, show agreement within 5%. It is shown that the limitations of the strain gages in measuring the notch root strains can account for a major part of the discrepancy. (Author)

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